A programming algorithm for a product of arithmetical operators on M-3.

Order code:

$0\Theta ab$:	$(b)\,\Theta\,(a) \Rightarrow (b),\ (R)$	(Θab)
$1\Theta ab$:	$(b)\,\Theta\,(a) \Rightarrow (R)$	(Θ, ab)
$2\Theta ab$:	$(R)\,\Theta\,(a) \Rightarrow (b),\ (R)$	$(\downarrow\Theta ab)$
$3\Theta ab$:	$(R)\,\Theta\,(a) \Rightarrow (R)$	$(\downarrow\Theta, ab)$
$4\Theta ab$:	$(b)\,\Theta\,(a) \Rightarrow (b),\ (R)$, paper	$(\Theta\pi\ ab)$
$5\Theta ab$:	$\lvert(b)\rvert\ \Theta\ \lvert(a)\rvert \Rightarrow (R)$	$(\lvert\Theta\rvert\ ab)$
$6\Theta ab$:	$(R)\,\Theta\,(a) \Rightarrow (b),\ (R)$, paper	$(\downarrow\Theta\pi\ ab)$
$7\Theta ab$:	$\lvert(R)\rvert\,\Theta\,\lvert(a)\rvert \Rightarrow (R)$	$(\downarrow\lvert\Theta\rvert\ ab)$
24 ab	$(R) \Rightarrow (b)$; next order (a)	(Пу a b)
34 ab	next order $\begin{cases}(a) \text{ if } (R) \text{ negative}\\ (b) \text{ if } (R) \text{ positive}\end{cases}$	(уП ab)
64 ab	$(R) \Rightarrow (b)$, paper; next order (a)	(ПуП ab
77 ab	stop	(Я ab)

$\Theta = +, -, :, \times, \wedge$
(code $\Theta = 0, 1, 2, 3, 6$)

Arithmetical terms

Complete bracketing

In fact not required

Hierarchy of elementary operations

Subterm

Non trivial subterm: not a variable

Different occurrences regarded as not identical subterms

Equality (formal)

Later on a stronger equality notion depending on place

Chain term or ~~ch~~chain

$(^n a_0 \theta_1 a_1)\theta_2 a_2)\theta_3 a_3)\cdots)\theta_n a_n)$

~~...~~ $(^n$ abbreviation for $((\cdots($ with n brackets

Order of a chain: n, $n \neq 0$

Chain operator: $c \Rightarrow x$, c a chain

Order same as that of c

Programming of a chain operator:

$(^n a_0 \theta_1 a_1)\theta_2 a_2)\theta_3 a_3)\cdots)\theta_n a_n) \Rightarrow x$

$1\theta_1$ a_1 a_0

$3\theta_2$ a_2

$3\theta_3$ a_3

$\cdots$

$3\theta_{n-1}$ a_{n-1}

$2\theta_n$ a_n x

In the case $n=1$, $(a_0 \theta a_1) \Rightarrow x$,

for $x \neq a_1$, 1θ a_1 a_0

$k.$ 24 $k{+}1$ x;

for $x = a_1$, 0θ a_1 a_0

Valid also for $\theta_i = |{+}|, |{-}|, |{:}|, |{\times}|, |\Lambda|$, provided

for $\theta = |\Lambda|$ we consider

as 0θ a b the orders 5Λ a b

$k.$ 24 $k{+}1$ b

as 1θ a b the order 5Λ a b

as 2θ a b the orders 7Λ a b

$k.$ 24 $k{+}1$ b

as 3θ a b the order $7\underline{\Lambda}$ a b

Problem of factorizing arithmetical operators into chain operators

Step by step, hence representation of a product of arithmetical operators as a product of chain operators

In the case of an 1 order chain operator, we intend to the identity of right hand side with first component on left hand side: $(a \ominus b) \Rightarrow a$

<u>Starting bracket</u>: left bracket not preceded by any symbol different from a left bracket

<u>Inner left bracket</u>: not starting

e.g. $((((x+y)z)-u):\underline{(}((x+y)z)+u)$

A term is a chain if and only if it does not contain any inner left bracket

<u>Inner (non trivial) subterm</u>: beginning with an inner left bracket

e.g. $(((x+y)z)+u)$ in the above example

<u>Block of left brackets</u>: no symbol different from a left bracket between

Let us given a ~~chain~~ product

$$P: \quad \underbrace{t_1 \Rightarrow x_1}_{O_1};\ \underbrace{t_2 \Rightarrow x_2}_{O_2};\ \cdots;\ \underbrace{t_n \Rightarrow x_n}_{O_n}$$

of arithmetical operators

<u>Starting, inner left bracket</u> in P: that in one of $t_1, t_2, \ldots, t_n$

<u>Inner subterm</u> of P: that of one of $t_1, t_2, \ldots, t_n$

<u>Maximal subchain</u> of P: not subterm of other subchain

Real equality of subterms of P: besides being formally equal, they do not contain a variable figuring between them on the right hand side of an operator

E.g. $\underline{(x+y)z}-u \Rightarrow v,\ \underline{(x+y)z}+u \Rightarrow w$ for real equality

$\underline{(x+y)z}-u \Rightarrow x,\ \underline{(x+y)z}+u \Rightarrow y$ for formal equality

$\underbrace{((x+y)z-u)}:v \Rightarrow u,\ \underbrace{((x+y)z+u)}:v \Rightarrow v,$

those underlined ⌴ formally, those underlined _ really equal

Example 1:

$((x-y)z+u):((x-y)z-u) \Rightarrow u;\ (((x+y)z-v)w-t):(((x+y)z-u)v+w) \Rightarrow w;$

$(((x+y)z-u)v-w):(((x+y)z-u)v+w) \Rightarrow v;$

$(((x+y)z-u)v+w):(((x+y)z-u)v-w) \Rightarrow t$

Steps: 1. Find the ~~left~~ maximal subchain $\underline{c}$ of P (═)

1,1 Find last left bracket;

1,2 Find the beginning of the block containing it;

1,3 Count the number of its left brackets

1,4 Find the same number of right brackets on the right of it

If c is on the beginning of P, then no factorization needed

2. Find the first subchain d of P which is really equal to some subchain of $\underline{c}$ (d=c allowed)

2,1 ~~Store~~ c in the memory;

2,2 Go to the left; when we find a proper subterm of c, replace c by it in the memory;

2,3 When we pass a variable x standing on the left hand side of ⇒, store also the maximal subterm of c not containing x, provided it is non trivial, otherwise do not go farther to the left

Example

$$(a+b+c)\tfrac{1}{2}\,((a+b+c)\tfrac{1}{2}-a)\,((a+b+c)\tfrac{1}{2}-b)\,((a+b+c)\tfrac{1}{2}-c)\Rightarrow x\ \ (=t^2);$$

$$\wedge\,\overline{(a+b+c)\tfrac{1}{2}\,\underline{((a+b+c)\tfrac{1}{2}-c)}:\underline{ab}}\Rightarrow y\ \ (=\cot\tfrac{5}{2})$$

$$\wedge\,\underline{(a+b+c)\tfrac{1}{2}}\,\underline{((a+b+c)\tfrac{1}{2}}-a)\,(\underline{(a+b+c)\tfrac{1}{2}}-b)\,\underline{((a+b+c)\tfrac{1}{2}}-c)\Rightarrow x;\ \ ab\Rightarrow y;$$
$$\underline{(a+b+c)\tfrac{1}{2}}\,\underline{((a+b+c)\tfrac{1}{2}-c)}:y\Rightarrow y$$

$$(a+b+c)\tfrac{1}{2}\Rightarrow s;\ \wedge\,\overline{s(s-a)(s-b)\underline{(s-c)}\Rightarrow x;\ ab\Rightarrow y;\ \underline{s(s-c)}:y\Rightarrow y}$$

$$(a+b+c)\tfrac{1}{2}\Rightarrow s;\ \ s-c\Rightarrow c;\ \wedge\,\overbrace{s(s-a)\underline{(s-b)}c}\Rightarrow x;\ ab\Rightarrow y;\ sc:y\Rightarrow y$$

$$(a+b+c)\tfrac{1}{2}\Rightarrow s;\ \ s-c\Rightarrow c;\ \ s-b\Rightarrow x,\ \ \overbrace{s\underline{(s-a)}xc}\Rightarrow x;\ ab\Rightarrow y;\ sc:y\Rightarrow y$$

$$(a+b+c)\tfrac{1}{2}\Rightarrow s;\ \ s-c\Rightarrow c;\ \ s-b\Rightarrow x;\ \ s-a\Rightarrow y;\ \ syxc\Rightarrow x;\ \ ab\Rightarrow y;\ \ sc:y\Rightarrow y$$

0.	1 + b	a	1+	22	21			14.	1×b	a	1×	22	21
1.	3 + c		3+	23	00			15.	24 16	y	24	16	25
2.	2×$\frac{1}{2}$	s	2×	26	27			16.	1×c	s	1×	23	27
3.	1 − c	s	1−	23	27			17.	2 : y	y	2 :	25	25
4.	24 5	c	24	5	23			20.	stop		stop		
5.	1 − b	s	1−	22	27			21.	a		a		
6.	24 7	x	24	7	24			22.	b		b		
7.	1 − a	s	1−	21	27			23.	c		c		
10.	24 11	y	24	11	25			24.	x		x		
11.	1×y	s	1×	25	27			25.	y		y		
12.	3×x		3×	24	00			26.	$\frac{1}{2}$		$\frac{1}{2}$		
13.	2×c	x	2×	23	24			27.	s		s		

Example 1:

$$((x+y)z-u) : ((x+y)z+u) \Rightarrow u; \; x : ((\underline{y+z})\,\underline{u}-v) \Rightarrow v;$$
$$y : ((\underline{y+z})\,\underline{u}-v) \Rightarrow w; \; ((x-y)z+u)v-w \Rightarrow t$$

$$((x+y)z-u) : ((x+y)z+u) \Rightarrow u; \; (y+z)u \Rightarrow r; \; x : (r-v) \Rightarrow v;$$
$$y : (r-v) \Rightarrow w; \; ((x-y)z+u\,v-w \Rightarrow t$$

Example 2:

$$((x+y)z+u) : z \Rightarrow y; \; ((\underline{x+y})z-u)v+w) : ((x-y)z+u) \Rightarrow z;$$
$$(((\underline{x+y})z-v)w-t) : (((\underline{x+y})\,\underline{z-u})v+w) \Rightarrow w;$$
$$(((\underline{x+y)z-u})v-w) : (((\underline{x+y)z-u})v+w) \Rightarrow v;$$
$$(\underline{((x+y)z-u)v}+w) : \underline{(((x+y)z-u)v-w)} \Rightarrow t$$

Computation of a determinant on M-3.

$$\begin{vmatrix} a_{11} \cdots a_{1n} \\ - - - - \\ a_{n1} \cdots a_{nn} \end{vmatrix} = 2^x y, \quad \tfrac{1}{2} \leq |y| < 1.$$

Method: elimination of all elements ~~on~~ ~~[illegible]~~ above the main diagonal by choosing the ~~[illegible]~~ elements with maximal absolute value

Choose a_{1j} with maximal $|a_{1j}|$; interchange ~~the~~ column j with column 1. Subtract first column, multiplyed by $\frac{a_{1j}}{a_{11}}$, from j^{th} column, however, by dividing the new elements by 2 and raising x by 1 each time. Supposing, the determinant is already transformed to

$$\begin{vmatrix}
a_{11} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
a_{21} & a_{22} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
- & - & - & & & & & & \\
a_{i-1,1} & a_{i-1,2} & a_{i-1,3} & \cdots & a_{i-1,i-1} & 0 & 0 & \cdots & 0 \\
a_{i1} & a_{i2} & a_{i3} & \cdots & a_{i,i-1} & a_{ii} & a_{i,i+1} & \cdots & a_{in} \\
a_{i+1,1} & a_{i+1,2} & a_{i+1,3} & \cdots & a_{i+1,i-1} & a_{i+1,i} & a_{i+1,i+1} & \cdots & a_{i+1,n} \\
- & - & - & - & - & - & - & & - \\
a_{n1} & a_{n2} & a_{n3} & \cdots & a_{n,i-1} & a_{ni} & a_{n,i+1} & \cdots & a_{nn}
\end{vmatrix},$$

choose $\max(|a_{ii}|, |a_{i,i+1}|, \ldots, |a_{in}|) = |a_{ij}|$, if $j \neq i$, interchange columns i and j (first $i-1$ elements not necessary), if after interchanging $a_{ii} = 0$, then take $x = y = 0$, otherwise subtract for $j = i+1, \ldots, n$, column i, multiplied by $\frac{a_{ij}}{a_{ii}}$, from column j, divide ~~by~~ new column j ~~by~~ 2 and raise x by 1. After doing this for $i = 1, 2, \ldots, n-1$, multiply diagonal elements and normalize y.

Computation of a determinant on M-3, operator scheme.

$$0 \Rightarrow x \;\Big\{_{i=1}^{n-1} \Big\{_{j=i+1}^{n} |a_{ii}|<|a_{ij}| \sqsubset_1 \Big\{_{k=i}^{n} a_{ki} \Rightarrow b;\ a_{kj} \Rightarrow a_{ki};\ -b \Rightarrow a_{kj} \Big\} \lrcorner_1 \Big]$$

$$a_{ii}=0 \sqsubset_2 0 \Rightarrow x;\ 0 \Rightarrow y \bullet \lrcorner_2 \Big\{_{j=i+1}^{n} \Big\{_{k=i}^{n} \tfrac{1}{2} a_{kj} - \tfrac{1}{2} a_{ki}\, a_{ij} : a_{ii} \Big\}\Big\}$$

$$x + n^* - i^* \Rightarrow x \Big\}\ a_{11} \Rightarrow y \Big\{_{i=2}^{n} y\, a_{ii} \Rightarrow y \Big\}$$

$$|y| < \tfrac{1}{2} \sqsubset_3 \Big\{_{i=1} y : \tfrac{1}{2} \Rightarrow y;\ x - 1^* \Rightarrow x \Big\}^{|y| \geq \frac{1}{2}} \lrcorner_3 \bullet$$

a_{ij} a_j 2

Transformation of

$$0 \Rightarrow x \Big\{_{i=0}^{n} 1 \Rightarrow y \Big\{_{j=i}^{n} y\, a_{ij} \Rightarrow y \Big\} x+y \Rightarrow x \Big\}:$$

$0 \Rightarrow j$

1) $$0 \Rightarrow x \Big\{_{i=0}^{n} 1 \Rightarrow y;\ i \Rightarrow j \lrcorner_1 y\, a_{ii} \Rightarrow y;\ j=n \sqsubset^2 \underline{j+1 \Rightarrow j};\ \Pi_2^1(\overset{1}{\Gamma}) \sqsubset_1^1$$

$$\lrcorner_2 \Pi_2^{-(n-i)}(\overset{1}{\Gamma});\ x+y \Rightarrow x \Big\}$$

2) $$0 \Rightarrow x \Big\{_{i=0}^{n} 1 \Rightarrow y;\ i \Rightarrow j \lrcorner_1 y\, a_{ii} \Rightarrow y;\ j=n \sqsubset^2 j+1 \Rightarrow j;\ \Pi_2^1(\overset{1}{\Gamma}) \sqsubset_1^1$$

$$\lrcorner_2 \underbrace{B(\overset{1}{\Gamma}, \overset{3}{\Gamma})}_{\overset{3}{\Gamma} \Rightarrow \overset{1}{\Gamma}};\ x+y \Rightarrow x \sqsubset_4^4 \lrcorner_3 y\, a_{ii} \Rightarrow y \lrcorner_4 \Big\}$$

$a_{11}\ a_{12}\ a_{13}\ a_1$

$$\overset{N}{\Gamma} \Big\{_{i=1} a_{i+1,j} \Rightarrow a_{ij} \Big\} \Rightarrow \overset{N}{\Gamma}$$

a_{ij}

$j \to j+1$

b_{ji}

$i \leq i \leq n$

$1 \leq j \leq n.$